47. *Finding f from f′* Let $f'(x) = 3x^2$.

(a) Compute the derivatives of $g(x) = x^3$, $h(x) = x^3 - 2$, and $t(x) = x^3 + 3$.

(b) Graph the numerical derivatives of g, h, and t.

(c) Describe a *family* of functions, $f(x)$, that have the property that $f'(x) = 3x^2$.

(d) Is there a function f such that $f'(x) = 3x^2$ and $f(0) = 0$? If so, what is it?

(e) Is there a function f such that $f'(x) = 3x^2$ and $f(0) = 3$? If so, what is it?

48. *Airplane Takeoff* Suppose that the distance an aircraft travels along a runway before takeoff is given by $D = (10/9)t^2$, where D is measured in meters from the starting point and t is measured in seconds from the time the brakes are released. If the aircraft will become airborne when its speed reaches 200 km/h, how long will it take to become airborne, and what distance will it have traveled by that time?

Extending the Ideas

49. *Even and Odd Functions*

(a) Show that if f is a differentiable even function, then f' is an odd function.

(b) Show that if f is a differentiable odd function, then f' is an even function.

50. *Extended Product Rule* Derive a formula for the derivative of the product fgh of three differentiable functions.

3.5 Derivatives of Trigonometric Functions

What you will learn about . . .

- Derivative of the Sine Function
- Derivative of the Cosine Function
- Simple Harmonic Motion
- Jerk
- Derivatives of the Other Basic Trigonometric Functions

and why . . .

The derivatives of sines and cosines play a key role in describing periodic change.

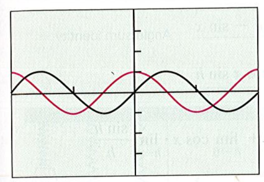
Derivative of the Sine Function

Trigonometric functions are important because so many of the phenomena we want information about are periodic (heart rhythms, earthquakes, tides, weather). It is known that continuous periodic functions can always be expressed in terms of sines and cosines, so the derivatives of sines and cosines play a key role in describing periodic change. This section introduces the derivatives of the six basic trigonometric functions.

EXPLORATION 1 Making a Conjecture by Graphing the Derivative

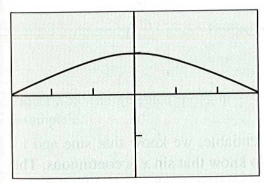
In the window $[-2\pi, 2\pi]$ by $[-4, 4]$, graph $y_1 = \sin x$ and $y_2 = \dfrac{d}{dx}(\sin x)$ (Figure 3.35). If your calculator does not recognize $\dfrac{d}{dx}(\sin x)$, use the numerical derivative.

1. When the graph of $y_1 = \sin x$ is increasing, what is true about the graph of $y_2 = \dfrac{d}{dx}(\sin x)$?
2. When the graph of $y_1 = \sin x$ is decreasing, what is true about the graph of $y_2 = \dfrac{d}{dx}(\sin x)$?
3. When the graph of $y_1 = \sin x$ stops increasing and starts decreasing, what is true about the graph of $y_2 = \dfrac{d}{dx}(\sin x)$?
4. At the places where $y_2 = \dfrac{d}{dx}(\sin x) = \pm 1$, what appears to be the slope of the graph of $y_1 = \sin x$?
5. Make a conjecture about what function the derivative of sine might be. Test your conjecture by graphing your function and $y_2 = \dfrac{d}{dx}(\sin x)$ in the same viewing window.
6. Now let $y_1 = \cos x$ and $y_2 = \dfrac{d}{dx}(\cos x)$. Answer questions (1) through (5) *without* looking at the graph of $y_2 = \dfrac{d}{dx}(\cos x)$ until you are ready to test your conjecture about what function the derivative of cosine might be.



$[-2\pi, 2\pi]$ by $[-4, 4]$

Figure 3.35 Sine and its derivative. What is the derivative? (Exploration 1)



$[-3, 3]$ by $[-2, 2]$

(a)

X	Y1
-.03	.99985
-.02	.99993
-.01	.99998
0	ERROR
.01	.99998
.02	.99993
.03	.99985

Y1 = sin(X)/X

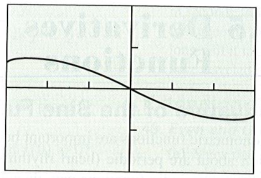
(b)

Figure 3.36 (a) Graphical and (b) tabular support that $\lim_{h \to 0} \dfrac{\sin(h)}{h} = 1$.

If you conjectured that the derivative of the sine function is the cosine function, then you are right. We will confirm this analytically, but first we appeal to technology one more time to evaluate two limits needed in the proof (see Figure 3.36 in the margin and Figure 3.37 on the next page):

A Word on Notation

In Exploration 1, a CAS calculator will accept the notation $y_2 = \frac{d}{dx}(\sin x)$ and graph the true derivative. Non-CAS users should use $y_2 =$ NDER(sin x, x), which works just as well as a visualization. In either case, you will be using your graph to make conjectures about the true derivative.



[−3, 3] by [−2, 2]

(a)

X	Y1
−.03	.015
−.02	.01
−.01	.005
0	ERROR
.01	−.005
.02	−.01
.03	−.015

Y1=(cos(X)−1)/X

(b)

Figure 3.37 (a) Graphical and (b) tabular support that $\lim_{h\to 0} \frac{\cos(h) - 1}{h} = 0.$

Confirm Analytically

(Also, see Section 2.1, Exercise 77.) Now, let $y = \sin x$. Then

$$\begin{aligned}
\frac{dy}{dx} &= \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} \\
&= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} \qquad \text{Angle sum identity} \\
&= \lim_{h\to 0} \frac{(\sin x)(\cos h - 1) + \cos x \sin h}{h} \\
&= \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{(\cos h - 1)}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h} \\
&= \sin x \cdot 0 + \cos x \cdot 1 \\
&= \cos x.
\end{aligned}$$

In short, the derivative of the sine is the cosine.

$$\frac{d}{dx}\sin x = \cos x$$

Now that we know that the sine function is differentiable, we know that sine and its derivative obey all the rules for differentiation. We also know that $\sin x$ is continuous. The same holds for the other trigonometric functions in this section. Each one is differentiable at every point in its domain, so each one is continuous at every point in its domain, and the differentiation rules apply for each one.

Derivative of the Cosine Function

If you conjectured in Exploration 1 that the derivative of the cosine function is the negative of the sine function, you were correct. You can confirm this analytically in Exercise 24.

$$\frac{d}{dx}\cos x = -\sin x$$

Radian Measure in Calculus

In case you have been wondering why calculus uses radian measure instead of degrees, you are now ready to understand the answer. The derivative of sin x is cos x *only* if x is measured in radians! If you look at the analytic confirmation, you will note that the derivative comes down to

$$\cos x \text{ times } \lim_{h\to 0}\frac{\sin h}{h}.$$

We saw that

$$\lim_{h\to 0}\frac{\sin h}{h} = 1$$

in Figure 3.36, but only because the graph in Figure 3.36 is in *radian mode*. If you look at the limit of the same function in *degree* mode you will get a very different limit (and hence a different derivative for sine). See Exercise 50.

EXAMPLE 1 Revisiting the Differentiation Rules

Find the derivatives of **(a)** $y = x^2 \sin x$ and **(b)** $u = \cos x/(1 - \sin x)$.

SOLUTION

(a)
$$\begin{aligned}
\frac{dy}{dx} &= x^2 \cdot \frac{d}{dx}(\sin x) + \sin x \cdot \frac{d}{dx}(x^2) \qquad \text{Product Rule}\\
&= x^2 \cos x + 2x \sin x
\end{aligned}$$

(b)
$$\begin{aligned}
\frac{du}{dx} &= \frac{(1-\sin x)\cdot \frac{d}{dx}(\cos x) - \cos x \cdot \frac{d}{dx}(1 - \sin x)}{(1-\sin x)^2} \qquad \text{Quotient Rule}\\
&= \frac{(1-\sin x)(-\sin x) - \cos x\,(0 - \cos x)}{(1-\sin x)^2}\\
&= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2}\\
&= \frac{1 - \sin x}{(1-\sin x)^2} \qquad \sin^2 x + \cos^2 x = 1\\
&= \frac{1}{1-\sin x}
\end{aligned}$$

Now Try Exercises 5 and 9.

Simple Harmonic Motion

The motion of a weight bobbing up and down on the end of a spring is an example of **simple harmonic motion.** Example 2 describes a case in which there are no opposing forces like friction or buoyancy to slow down the motion.

EXAMPLE 2 The Motion of a Weight on a Spring

A weight hanging from a spring (Figure 3.38) is stretched 5 units beyond its rest position ($s = 0$) and released at time $t = 0$ to bob up and down. Its position at any later time t is

$$s = 5 \cos t.$$

What are its velocity and acceleration at time t? Describe its motion.

SOLUTION We have:

Position: $s = 5\cos t$;

Velocity: $v = \frac{ds}{dt} = \frac{d}{dt}(5\cos t) = -5\sin t$;

Acceleration: $a = \frac{dv}{dt} = \frac{d}{dt}(-5\sin t) = -5\cos t.$

Notice how much we can learn from these equations:

1. As time passes, the weight moves down and up between $s = -5$ and $s = 5$ on the s-axis. The amplitude of the motion is 5. The period of the motion is 2π.
2. The velocity $v = -5\sin t$ attains its greatest magnitude, 5, when $\cos t = 0$, as the graphs show in Figure 3.39. Hence the speed of the weight, $|v| = 5|\sin t|$, is greatest when $\cos t = 0$, that is, when $s = 0$ (the rest position). The speed of the weight is zero when $\sin t = 0$. This occurs when $s = 5\cos t = \pm 5$, at the endpoints of the interval of motion.
3. The acceleration value is always the exact opposite of the position value. When the weight is above the rest position, gravity is pulling it back down; when the weight is below the rest position, the spring is pulling it back up.

continued

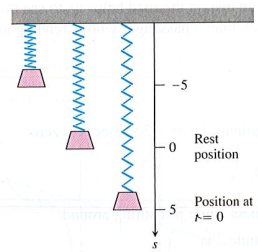


Figure 3.38 The weighted spring in Example 2.

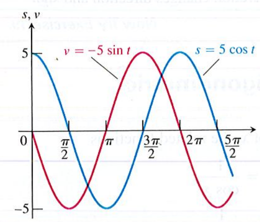


Figure 3.39 Graphs of the position and velocity of the weight in Example 2.

4. The acceleration, $a = -5\cos t$, is zero only at the rest position where $\cos t = 0$ and the force of gravity and the force from the spring offset each other. When the weight is anywhere else, the two forces are unequal and acceleration is nonzero. The acceleration is greatest in magnitude at the points farthest from the rest position, where $\cos t = \pm 1$.

Now Try Exercise 11.

Jerk

A sudden change in acceleration is called a "jerk." When a ride in a car or a bus is jerky, it is not that the accelerations involved are necessarily large but that the changes in acceleration are abrupt. Jerk is what spills your soft drink. The derivative responsible for jerk is the *third* derivative of position.

DEFINITION Jerk

Jerk is the derivative of acceleration. If a body's position at time t is $s(t)$, the body's jerk at time t is

$$j(t) = \frac{da}{dt} = \frac{d^3s}{dt^3}.$$

Tests have shown that motion sickness comes from accelerations whose changes in magnitude or direction take us by surprise. Keeping an eye on the road helps us to see the changes coming. A driver is less likely to become sick than a passenger who is reading in the back seat.

EXAMPLE 3 A Couple of Jerks

(a) The jerk caused by the constant acceleration of gravity ($g = -32$ ft/sec^2) is zero:

$$j = \frac{d}{dt}(g) = 0.$$

This explains why we don't experience motion sickness while just sitting around.

(b) The jerk of the simple harmonic motion in Example 2 is

$$j = \frac{da}{dt} = \frac{d}{dt}(-5\cos t)$$
$$= 5\sin t.$$

It has its greatest magnitude when $\sin t = \pm 1$. This does not occur at the extremes of the displacement, but at the rest position, where the acceleration changes direction and sign.

Now Try Exercise 19.

Derivatives of the Other Basic Trigonometric Functions

Because $\sin x$ and $\cos x$ are differentiable functions of x, the related functions

$$\tan x = \frac{\sin x}{\cos x}, \qquad \sec x = \frac{1}{\cos x},$$

$$\cot x = \frac{\cos x}{\sin x}, \qquad \csc x = \frac{1}{\sin x}$$

are differentiable at every value of x for which they are defined. Their derivatives (Exercises 25 and 26) are given by the following formulas.

$$\frac{d}{dx}\tan x = \sec^2 x, \qquad \frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\cot x = -\csc^2 x, \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

EXAMPLE 4 Finding Tangent and Normal Lines

Find equations for the lines that are tangent and normal to the graph of

$$f(x) = \frac{\tan x}{x}$$

at $x = 2$. Support graphically.

SOLUTION

Solve Numerically Since we will be using a calculator approximation for $f(2)$ anyway, this is a good place to use NDER.

We compute $(\tan 2)/2$ on the calculator and store it as k. The slope of the tangent line at $(2, k)$ is

$$\text{NDER}\left(\frac{\tan x}{x}, 2\right),$$

which we compute and store as m. The equation of the tangent line is $y - k = m(x - 2)$, or

$$y = mx + k - 2m.$$

Only after we have found m and $k - 2m$ do we round the coefficients, giving the tangent line as

$$y = 3.43x - 7.96.$$

The equation of the normal line is

$$y - k = -\frac{1}{m}(x - 2), \text{ or}$$

$$y = -\frac{1}{m}x + k + \frac{2}{m}.$$

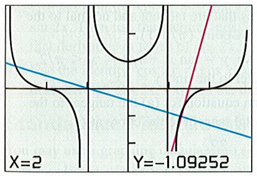
Again we wait until the end to round the coefficients, giving the normal line as

$$y = -0.291x - 0.51.$$

Support Graphically Figure 3.40, showing the original function and the two lines, supports our computations.

Now Try Exercise 23.

$y_1 = \tan(x)/x$
$y_2 = 3.43x - 7.96$
$y_3 = -0.291x - 0.51$



$[-3\pi/2, 3\pi/2]$ by $[-3, 3]$

Figure 3.40 Graphical support for Example 4.

EXAMPLE 5 A Trigonometric Second Derivative

Find y'' if $y = \sec x$.

SOLUTION

$$y = \sec x$$
$$y' = \sec x \tan x$$
$$y'' = \frac{d}{dx}(\sec x \tan x)$$
$$= \sec x \frac{d}{dx}(\tan x) + \tan x \frac{d}{dx}(\sec x)$$
$$= \sec x(\sec^2 x) + \tan x(\sec x \tan x)$$
$$= \sec^3 x + \sec x \tan^2 x$$

Now Try Exercise 36.

Quick Review 3.5 (For help, go to Sections 1.6, 3.1, and 3.4.)

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator.*

1. Convert 135 degrees to radians.
2. Convert 1.7 radians to degrees.
3. Find the exact value of $\sin(\pi/3)$ without a calculator.
4. State the domain and the range of the cosine function.
5. State the domain and the range of the tangent function.
6. If $\sin a = -1$, what is $\cos a$?
7. If $\tan a = -1$, what are two possible values of $\sin a$?
8. Verify the identity:
$$\frac{1-\cos h}{h} = \frac{\sin^2 h}{h(1+\cos h)}.$$
9. Find an equation of the line tangent to the curve $y = 2x^3 - 7x^2 + 10$ at the point $(3, 1)$.
10. A particle moves along a line with velocity $v = 2t^3 - 7t^2 + 10$ for time $t \ge 0$. Find the acceleration of the particle at $t = 3$.

Section 3.5 Exercises

In Exercises 1–10, find dy/dx. Use your grapher to support your analysis if you are unsure of your answer.

1. $y = 1 + x - \cos x$
2. $y = 2\sin x - \tan x$
3. $y = \dfrac{1}{x} + 5\sin x$
4. $y = x\sec x$
5. $y = 4 - x^2\sin x$
6. $y = 3x + x\tan x$
7. $y = \dfrac{4}{\cos x}$
8. $y = \dfrac{x}{1+\cos x}$
9. $y = \dfrac{\cot x}{1+\cot x}$
10. $y = \dfrac{\cos x}{1+\sin x}$

In Exercises 11 and 12, a weight hanging from a spring (see Figure 3.38) bobs up and down with position function $s = f(t)$ (s in meters, t in seconds). What are its velocity and acceleration at time t? Describe its motion.

11. $s = 5\sin t$
12. $s = 7\cos t$

In Exercises 13–16, a body is moving in simple harmonic motion with position function $s = f(t)$ (s in meters, t in seconds).

(a) Find the body's velocity, speed, and acceleration at time t.

(b) Find the body's velocity, speed, and acceleration at time $t = \pi/4$.

(c) Describe the motion of the body.

13. $s = 2 + 3\sin t$
14. $s = 1 - 4\cos t$
15. $s = 2\sin t + 3\cos t$
16. $s = \cos t - 3\sin t$

In Exercises 17–20, a body is moving in simple harmonic motion with position function $s = f(t)$ (s in meters, t in seconds). Find the jerk at time t.

17. $s = 2\cos t$
18. $s = 1 + 2\cos t$
19. $s = \sin t - \cos t$
20. $s = 2 + 2\sin t$
21. Find equations for the lines that are tangent and normal to the graph of $y = \sin x + 3$ at $x = \pi$.
22. Find equations for the lines that are tangent and normal to the graph of $y = \sec x$ at $x = \pi/4$.
23. Find equations for the lines that are tangent and normal to the graph of $y = x^2\sin x$ at $x = 3$.
24. Use the definition of the derivative to prove that $(d/dx)(\cos x) = -\sin x$. (You will need the limits found at the beginning of this section.)
25. Assuming that $(d/dx)(\sin x) = \cos x$ and $(d/dx)(\cos x) = -\sin x$, prove each of the following.

(a) $\dfrac{d}{dx}\tan x = \sec^2 x$ (b) $\dfrac{d}{dx}\sec x = \sec x \tan x$

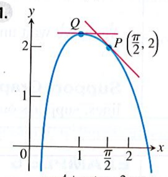
26. Assuming that $(d/dx)(\sin x) = \cos x$ and $(d/dx)(\cos x) = -\sin x$, prove each of the following.

(a) $\dfrac{d}{dx}\cot x = -\csc^2 x$ (b) $\dfrac{d}{dx}\csc x = -\csc x \cot x$

27. Show that the graphs of $y = \sec x$ and $y = \cos x$ have horizontal tangents at $x = 0$.
28. Show that the graphs of $y = \tan x$ and $y = \cot x$ have no horizontal tangents.
29. Find equations for the lines that are tangent and normal to the curve $y = \sqrt{2}\cos x$ at the point $(\pi/4, 1)$.
30. Find the points on the curve $y = \tan x$, $-\pi/2 < x < \pi/2$, where the tangent is parallel to the line $y = 2x$.

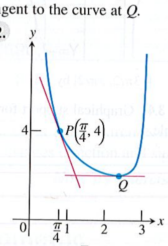
In Exercises 31 and 32, find an equation for (a) the tangent to the curve at P and (b) the horizontal tangent to the curve at Q.

31.



$y = 4 + \cot x - 2\csc x$

32.



$y = 1 + \sqrt{2}\csc x + \cot x$

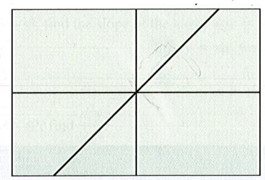
Group Activity In Exercises 33 and 34, a body is moving in simple harmonic motion with position $s = f(t)$ (s in meters, t in seconds).

(a) Find the body's velocity, speed, acceleration, and jerk at time t.

(b) Find the body's velocity, speed, acceleration, and jerk at time $t = \pi/4$ sec.

(c) Describe the motion of the body.

33. $s = 2 - 2\sin t$
34. $s = \sin t + \cos t$
35. Find y'' if $y = \csc x$.
36. Find y'' if $y = \theta\tan\theta$.
37. **Writing to Learn** Is there a value of b that will make
$$g(x) = \begin{cases} x + b, & x < 0 \\ \cos x, & x \ge 0 \end{cases}$$
continuous at $x = 0$? differentiable at $x = 0$? Give reasons for your answers.
38. Find $\dfrac{d^{999}}{dx^{999}}(\cos x)$.
39. Find $\dfrac{d^{725}}{dx^{725}}(\sin x)$.
40. ***Local Linearity*** This is the graph of the function $y = \sin x$ close to the origin. Since $\sin x$ is differentiable, this graph resembles a line. Find an equation for this line.



41. ***(Continuation of Exercise 40)*** For values of x close to 0, the linear equation found in Exercise 40 gives a good approximation of $\sin x$.

(a) Use this fact to estimate $\sin(0.12)$.

(b) Find $\sin(0.12)$ with a calculator. How close is the approximation in part (a)?

42. Use the identity $\sin 2x = 2\sin x\cos x$ to find the derivative of $\sin 2x$. Then use the identity $\cos 2x = \cos^2 x - \sin^2 x$ to express that derivative in terms of $\cos 2x$.
43. Use the identity $\cos 2x = \cos x\cos x - \sin x\sin x$ to find the derivative of $\cos 2x$. Express the derivative in terms of $\sin 2x$.

Standardized Test Questions

You may use a graphing calculator to solve the following problems.

In Exercises 44 and 45, a spring is bobbing up and down on the end of a spring according to $s(t) = -3\sin t$.

44. **True or False** The spring is traveling upward at $t = 3\pi/4$. Justify your answer.
45. **True or False** The velocity and speed of the particle are the same at $t = \pi/4$. Justify your answer.
46. **Multiple Choice** Which of the following is an equation of the tangent line to $y = \sin x + \cos x$ at $x = \pi$?

(A) $y = -x + \pi - 1$ (B) $y = -x + \pi + 1$

(C) $y = -x - \pi + 1$ (D) $y = -x - \pi - 1$

(E) $y = x - \pi + 1$

47. **Multiple Choice** Which of the following is an equation of the normal line to $y = \sin x + \cos x$ at $x = \pi$?

(A) $y = -x + \pi - 1$ (B) $y = x - \pi - 1$

(C) $y = x - \pi + 1$ (D) $y = x + \pi + 1$

(E) $y = x + \pi - 1$

48. **Multiple Choice** Find y'' if $y = x\sin x$.

(A) $-x\sin x$ (B) $x\cos x + \sin x$ (C) $-x\sin x + 2\cos x$

(D) $x\sin x$ (E) $-\sin x + \cos x$

49. **Multiple Choice** A body is moving in simple harmonic motion with position $s = 3 + \sin t$. At which of the following times is the velocity zero?

(A) $t = 0$ (B) $t = \pi/4$ (C) $t = \pi/2$

(D) $t = \pi$ (E) none of these

Exploration

50. ***Radians vs. Degrees*** What happens to the derivatives of $\sin x$ and $\cos x$ if x is measured in degrees instead of radians? To find out, take the following steps.

(a) With your grapher in degree mode, graph
$$f(h) = \frac{\sin h}{h}$$
and estimate $\lim_{h\to 0} f(h)$. Compare your estimate with $\pi/180$. Is there any reason to believe the limit should be $\pi/180$?

(b) With your grapher in degree mode, estimate
$$\lim_{h\to 0}\frac{\cos h - 1}{h}.$$

(c) Now go back to the derivation of the formula for the derivative of $\sin x$ in the text and carry out the steps of the derivation using degree-mode limits. What formula do you obtain for the derivative?

(d) Derive the formula for the derivative of $\cos x$ using degree-mode limits.

(e) The disadvantages of the degree-mode formulas become apparent as you start taking derivatives of higher order. What are the second and third degree-mode derivatives of $\sin x$ and $\cos x$?

Extending the Ideas

51. Use analytic methods to show that
$$\lim_{h\to 0}\frac{\cos h - 1}{h} = 0.$$
[*Hint:* Multiply numerator and denominator by $(\cos h + 1)$.]
52. Find A and B in $y = A\sin x + B\cos x$ so that $y'' - y = \sin x$.

Quick Quiz for AP* Preparation: Sections 3.4–3.5

1. Multiple Choice If the line tangent to the graph of the function f at the point $(1, 6)$ passes through the point $(-1, -4)$, then $f'(1)$ is

(A) -1 (B) 1 (C) 5 (D) 6 (E) undefined

2. Multiple Choice Which of the following gives y'' for $y = \cos x + \tan x$?

(A) $-\cos x + 2\sec^2 x \tan x$ (B) $\cos x + 2\sec^2 x \tan x$

(C) $-\sin x + \sec^2 x$ (D) $-\cos x + \sec^2 x \tan x$

(E) $\cos x + \sec^2 x \tan x$

3. Multiple Choice If $y = \dfrac{3x + 2}{2x + 3}$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{12x - 13}{(2x + 3)^2}$ (B) $\dfrac{12x - 13}{(2x + 3)^2}$ (C) $\dfrac{-5}{(2x + 3)^2}$

(D) $\dfrac{5}{(2x + 3)^2}$ (E) $\dfrac{2}{3}$

4. Free Response A particle moves along a line so that its position at any time $t \ge 0$ is given by $s(t) = -t^2 + t + 2$, where s is measured in meters and t is measured in seconds.

(a) What is the initial position of the particle?

(b) Find the initial velocity of the particle at any time t.

(c) When is the particle moving to the right?

(d) Find the acceleration of the particle at any time t.

(e) Find the speed of the particle at the moment when $s(t) = 0$.

Chapter 3 Key Terms

acceleration (p. 130)
average velocity (p. 128)
Constant Multiple Rule (p. 117)
Derivative of a Constant Function (p. 116)
derivative of f at a (p. 99)
differentiable function (p. 99)
differentiable on a closed interval (p. 104)
displacement (p. 128)
free-fall constants (p. 130)
instantaneous rate of change (p. 127)
instantaneous velocity (p. 128)
Intermediate Value Theorem for Derivatives (p. 113)
jerk (p. 144)
left-hand derivative (p. 104)
local linearity (p. 110)
marginal cost (p. 134)
marginal revenue (p. 134)
nth derivative (p. 122)
numerical derivative (NDER) (p. 111)
Power Rule for Negative Integer Powers of x (p. 121)
Power Rule for Positive Integer Powers of x (p. 116)
Product Rule (p. 119)
Quotient Rule (p. 120)
right-hand derivative (p. 104)
sensitivity to change (p. 133)
simple harmonic motion (p. 143)
speed (p. 129)
Sum and Difference Rule (p. 117)
symmetric difference quotient (p. 111)
velocity (p. 128)

Chapter 3 Review Exercises

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator.*

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1–30, find the derivative of the function.

1. $y = x^5 - \frac{1}{8}x^2 + \frac{1}{4}x$ **2.** $y = 3 - 7x^3 + 3x^7$

3. $y = 2 \sin x \cos x$ **4.** $y = \dfrac{2x + 1}{2x - 1}$

5. $s = (t^2 - 1)(t^2 + 1)$ **6.** $s = \dfrac{t^2 + 1}{1 - t^2}$

7. $y = \sqrt{x} + 1 + \dfrac{1}{\sqrt{x}}$ **8.** $y = (x^5 + 1)(3x^2 - x)$

9. $r = 5\theta^2 \sec \theta$ **10.** $r = \dfrac{\tan \theta}{\theta^3 + \theta + 1}$

11. $y = x^2 \sin x + x \cos x$ **12.** $y = x^2 \sin x - x \cos x$

13. $y = \dfrac{\tan x}{2x^3}$ **14.** $y = \tan x - \cot x$

15. $y = \dfrac{1}{\sin x + \cos x}$ **16.** $y = \dfrac{1}{\sin x} + \dfrac{1}{\cos x}$

17. $V = \frac{4}{3}\pi r^3 + 8\pi r^2$ **18.** $A = \dfrac{\sqrt{3}}{4}s^2 + \dfrac{3\pi}{8}s^2$

19. $s = \dfrac{1 + \sin t}{1 + \tan t}$ **20.** $s = \dfrac{1 + \sin t}{1 + \cos t}$

21. $s = \dfrac{t^{-1} + t^{-2}}{t^{-3}}$ **22.** $y = x^{-2} \cos x - 4x^{-3}$

23. $y = \dfrac{\sin u}{\csc u} + \dfrac{\cos u}{\sec u}$ **24.** $y = \dfrac{\cot u}{\tan u} - \dfrac{\csc u}{\sin u}$

25. $y = 2x^{-2}(x^5 - x^3)$ **26.** $y = 4x^2(x^{-1} + 3x^{-4})$

27. $y = \dfrac{t^2}{\pi^3} - \dfrac{\pi^2}{t^3}$ **28.** $y = \dfrac{t^3}{\pi^2} - \dfrac{\pi^3}{t^2}$

29. $y = \sec x \tan x \cos x$ **30.** $y = \dfrac{\sin x \cot x}{\cos x}$

In Exercises 31–34, find all values of x for which the function is differentiable.

31. $y = \dfrac{\sin x}{x}$ **32.** $y = \sin x - x \cos x$

33. $y = \dfrac{3 \cos x}{x - 2}$ **34.** $y = (2x - 7)^{-1}(x + 5)$

In Exercises 35–38, find the slope of the curve at $x = \pi$.

35. $y = \sec x$ **36.** $y = \sin x \cos x$

37. $y = \dfrac{\cos x}{x}$ **38.** $y = \dfrac{x}{x + \sin x}$

In Exercises 39–42, find $\dfrac{d^2y}{dx^2}$.

39. $y = \dfrac{1}{\cos x}$ **40.** $y = \csc x$

41. $y = x \sin x$ **42.** $y = x - x \cos x$

In Exercises 43 and 44, find all derivatives of the function.

43. $y = \dfrac{x^4}{2} - \dfrac{3}{2}x^2 - x$ **44.** $y = \dfrac{x^5}{120}$

In Exercises 45–48, find an equation for the **(a)** tangent and **(b)** normal to the curve at the indicated point.

45. $y = 8x^{-2}, \quad x = 2$

46. $y = 4 + \cot x - 2 \csc x, \quad x = \pi/2$

47. $y = \sin x + \cos x, \quad x = \dfrac{\pi}{4}$ **48.** $y = 2x^2 + \dfrac{1}{x^4}, \quad x = 1$

In Exercises 49–52, find the coordinates of all points on the curve at which the tangent line has slope 6. If no such point exists, write "none."

49. $y = 2x^3$ **50.** $y = \dfrac{2x^3 - 3x^2}{6}$

51. $y = \dfrac{6x}{x + 1}$ **52.** $y = 2 \sin x$

53. Writing to Learn

(a) Graph the function

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2. \end{cases}$$

(b) Is f continuous at $x = 1$? Explain.

(c) Is f differentiable at $x = 1$? Explain.

54. Writing to Learn For what values of the constant m is

$$f(x) = \begin{cases} 2\sin x, & x \le 0 \\ mx, & x > 0 \end{cases}$$

(a) continuous at $x = 0$? Explain.

(b) differentiable at $x = 0$? Explain.

In Exercises 55–58, determine where the function is **(a)** differentiable, **(b)** continuous but not differentiable, and **(c)** neither continuous nor differentiable.

55. $f(x) = x^{4/5}$

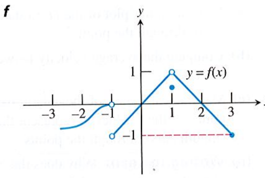
56. $y = x^{3/5}$

57. $f(x) = \begin{cases} 2x - 3, & -1 \le x < 0 \\ x - 3, & 0 \le x \le 4 \end{cases}$

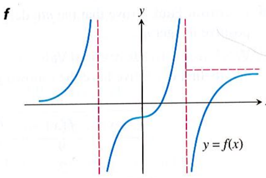
58. $g(x) = \begin{cases} \dfrac{x - 1}{x}, & -2 \le x < 0 \\ \dfrac{x + 1}{x}, & 0 \le x \le 2 \end{cases}$

In Exercises 59 and 60, use the graph of f to sketch the graph of f'.

59. *Sketching f′ from f*

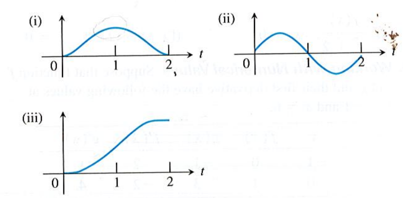


60. *Sketching f′ from f*



61. *Recognizing Graphs* The following graphs show the distance traveled, velocity, and acceleration for each second of a 2-minute automobile trip. Which graph shows

(a) distance? **(b)** velocity? **(c)** acceleration?



62. *Sketching f from f′* Sketch the graph of a continuous function f with $f(0) = 5$ and

$$f'(x) = \begin{cases} -2, & x < 2 \\ -0.5, & x > 2. \end{cases}$$

63. *Sketching f from f′* Sketch the graph of a continuous function f with $f(-1) = 2$ and

$$f'(x) = \begin{cases} -2, & x < 1 \\ 1, & 1 < x < 4 \\ -1, & 4 < x < 6. \end{cases}$$